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HYDROGASDYNAMICS AND HEAT TRANSFER IN TECHNOLOGICAL PROCESSES

TRANSFER OF DISCRETE INCLUSIONS BY FLUXES WITH CONCENTRATED VORTICITY

K. N. Volkov

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Problems related to modeling of the motion of discrete inclusions (solid particles, drops or bubbles) in flows with concentrated vorticity are considered. A comparative evaluation of the force factors in the equation of motion of a test particle is made. The results of numerical modeling of the motion of discrete inclusions in the gap between concentric rotating cylinders and a vortex flow formed by the liquid rotating with a constant angular velocity over a fixed base are discussed. The coordinates of the points of equilibrium of the test particle in the vortex flow are found.

Introduction. Application of devices (filters, burners, vortex reactors, cyclone separators) based on the use of vortex effects allows much room for intensifying a number of processes (mixing, combustion) and managing their stability. In particular, the favorable effect of twisting of an injected mixture of a fuel with an oxidizer is used for stabilizing intensive processes of combustion and organization of effective pure burning in many industrial installations. The perfecting of vertex facilities constitutes one of the problems on introducing energy- and resources-saving technologies into engineering practice. In many cases, the enhancement of the transfer properties of a medium is associated with the availability of discrete inclusions (solid particles, drops or bubbles).

In calculating two-phase flows, one of the principal questions is that of constructing a model of interaction of an individual particle, drop, or bubble with a liquid or gas flow.

The investigation of the force factors that exert their influence on the motion of discrete inclusions in vortex flows is the concern of a rather large number of publications [1–7]. The main contribution to the interphase interaction is made by the force of hydrodynamic resistance. In addition to the resistance force, the transfer of discrete inclusions by a vortex flow is influenced also by other factors related to the change in the velocity and acceleration in the relative motion of a particle and a liquid, including the force of the associated mass and the buoyancy force, as well as external body forces [1–4].

Despite the fact that relationships for calculating the forces acting on a particle, drop, or bubble are well known [8, 9] (including those with various corrections, for example, for inertia and internal circulation motion of a liquid inside a drop), the justification of taking or not taking account of any force factors requires additional investigation with allowance for the conditions of a specific problem.

In the present work, an evaluation of force factors is made, and modeling of the motion of discrete inclusions in flows with concentrated vorticity is considered. The results of calculations of the motion of discrete inclusions in a vortex flow originating between concentric rotating cylinders, as well as their motion in a rotating flow of liquid over a fixed plane, are discussed.

Types of Vortex Flows. To classify vortex flows, we will consider the motion of a liquid in a cylindrical coordinate system (x, r, φ). The coordinate x is reckoned along the axis of rotation, whereas the coordinates r and φ have their origin in the plane in which the cross-flow motion of the liquid occurs.

D. F. Ustinov "VOENMEKH" Baltic State Technical University, 1 1st Krasnoarmeiskaya Str., St. Petersburg, 190005, Russia; email: dsci@mail.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 80, No. 2, pp. 36–45, March–April, 2007. Original article submitted July 29, 2005.

TABLE 1. Characteristics of Different Types of Vortices

Flow characteristics	Free vortex	Forced vortex	Combined vortex
Tangential velocity u_φ	c/r	cr	$\frac{c}{r}\left[1-\exp\left(-\frac{r^2}{r_0^2}\right)\right]$
Angular velocity ω	c/r^2	c	$\psi(r)$
Circulation Γ	$2\pi c$	$2\pi\Omega r^2$	$2\pi c\left[1-\exp\left(-\frac{r^2}{r_0^2}\right)\right]$
Vorticity Ω_x	0	$4\pi\Omega$	$\frac{4\pi c}{r_0^2}\left[1-\exp\left(-\frac{r^2}{r_0^2}\right)\right]$

The vortex flows are characterized by the vorticity vector $\mathbf{\Omega} = \nabla \times \mathbf{u}$, angular speed of rotation of the liquid particle near the rotation axis u_φ / r, as well as by circulation around closed path $\mathbf{l}$:

$$\Gamma = \oint_l \mathbf{u} \cdot d\mathbf{l} \,,$$

where $d\mathbf{l}$ is an infinitely small increment of $\mathbf{l}$. Let us assume axial symmetry and vanishing axial and radial velocity components: $u_x = u_r = 0$. The circumferential velocity depends only on the radial coordinate $u_\varphi = u_\varphi(r)$. In this case, there is only one nonzero component of vorticity in the axial direction:

$$\Omega_x = \frac{1}{r} \frac{\partial r u_\varphi}{\partial r} \,.$$

Depending on the change in the tangential velocity along the radial coordinate, three types of axisymmetrical vortex flows are distinguished [10, 11], the characteristics of which are given in Table 1.

The vorticity in free vortices is equal to zero (the flow is potential), and the liquid particles follow along the streamlines, which represent concentric circles. With distance from the rotation axis, the tangential velocity tends to zero. The circulation is constant in magnitude. The forced vortices have a nonzero constant vorticity and angular frequency, whereas the tangential velocity and circulation increase with distance from the rotation axis. The free and forced vortices differ in the radial position of the maximum of the circumferential velocity. In a free vortex, the maximum is located near the symmetry axis and in a forced one — on its outer boundary.

In a combined vortex there is a vortex core of radius r_0. With increase in the radial coordinate, the tangential velocity inside the core is increased and is decreased beyond it:

$$u_\varphi \approx \frac{1}{r}\left[1-\exp\left(-\frac{r^2}{r_0^2}\right)\right].$$

The characteristics of a free-forced vortex are specified by the expressions for a forced vortex at small r's (for $r \to 0$) and by the expressions for a free vortex at high r's (for $r \to \infty$).

The equation of vorticity transfer has the form [10]

$$\frac{\partial \mathbf{\Omega}}{\partial t} + (\mathbf{u} \cdot \nabla)\, \mathbf{\Omega} = -\nabla \frac{1}{\rho} \times \nabla p + \nabla \times \nabla \sigma - \mathbf{\Omega}\, (\nabla \cdot \mathbf{u}) + (\mathbf{\Omega} \cdot \nabla)\, \mathbf{u} \,. \tag{1}$$

The terms on the right-hand side of Eq. (1) describe the baroclinic moment, viscous dissipation, expansion, and extension of vortex tubes. An increase in the flow acceleration and a decrease in viscous dissipation lead to the development of a thinner and more intense vortex core whose radius is given by the relation

$$r_0 = \left(\frac{4\nu}{\partial u_x / \partial x} \right)^{1/2} .$$

An analysis of various terms in Eq. (1) shows that the main mechanism underlying the concentration of vorticity is associated with extension of vortex tubes [10, 11].

Forces Acting on the Particle. The motion of a particle of a spherical shape is described by the equation

$$m_{\rm p} \frac{d\mathbf{v}_{\rm p}}{dt} = \mathbf{f}_D + \mathbf{f}_L + \mathbf{f}_m + \mathbf{f}_{\rm A} + \mathbf{f}_{\rm B} + \mathbf{f}_g + \mathbf{f}_{\rm b} .$$

The force acting on the particle is represented as a sum of the resistance force $\mathbf{f}_D$ (of the Stokesian resistance force at a low relative velocity of the particle and liquid), the buoyancy force $\mathbf{f}_L$ originating as a result of the motion of a particle along the circular trajectory, the force of associated mass $\mathbf{f}_m$, the Archimedes force $\mathbf{f}_{\rm A}$ associated with the pressure gradient, and the hereditary Bassé force $\mathbf{f}_{\rm B}$. In addition to the indicated force factors the particle experiences the action of the gravity force and buoyancy force:

$$\mathbf{f}_g + \mathbf{f}_{\rm b} = (\rho_{\rm p} - \rho)\, V_{\rm p} \mathbf{g} .$$

The force of hydrodynamic resistance can be found from the relation

$$\mathbf{f}_D = \frac{1}{2} C_D \, \rho \, |\mathbf{u} - \mathbf{v}| \, (\mathbf{u} - \mathbf{v}) \, S ,$$

where S is the area of the midsection of the particle (for a sphere $S = \pi r_{\rm p}^2$). The resistance coefficient is represented as

$$C_D = \frac{16}{{\rm Re}_{\rm p}} \psi_1 ({\rm Re}_{\rm p}) \, \psi_2 \left(\frac{\mu}{\mu_{\rm p}} \right).$$

The function ψ_1 takes into account the correction for the particle inertia. The Reynolds number is calculated from the relative velocity of the motion of the particle and liquid:

$${\rm Re}_{\rm p} = \frac{2 r_{\rm p} \, \rho \, |\mathbf{u} - \mathbf{v}|}{\mu} .$$

The function ψ_2 takes into account the internal circulation motion of a liquid inside a drop; it weakens the friction on its surface [8]:

$$\psi_2 = \frac{\mu + 3\mu_{\rm p}/2}{\mu + \mu_{\rm p}} .$$

The case $\mu_{\rm p}/\mu \to \infty$ corresponds to the Stokes law for a solid sphere $C_D = 24/{\rm Re}_{\rm p}$, whereas $\mu_{\rm p}/\mu \to 0$ — to the flow past a liquid drop for which $C_D = 16/{\rm Re}_{\rm p}$. For a gas bubble, when its spherical shape is preserved, the resistance law is described by the formula $C_D = 48/{\rm Re}_{\rm p}$.

The buoyancy force originating during the motion of the particle along the circular trajectory tends to displace the particles into the region with a lowered pressure (the region with a higher velocity), and for a spherical particle it is represented in the form [4]

$$\mathbf{f}_L = C_L \, \rho V_{\rm p} \, (\mathbf{u} - \mathbf{v}) \times \mathbf{\Omega} .$$

At high Reynolds numbers the value $C_L = 1/2$ is used [4, 7]. The dependence of the coefficient C_L on the magnitude of vorticity is rather weak [5].

The force of the associated mass takes into account the increase in the inertia of the particle moving with a variable velocity due to the necessity of setting in motion a certain mass of the liquid adjacent to the particle surface. This additional motion of the liquid is equivalent to the motion of the fictitious mass equal to half the mass of the liquid displaced by the sphere and moving with the same relative velocity as the particle. The force of the associated mass is given by the relation

$$\mathbf{f}_m = C_m \rho V_\mathrm{p} \frac{d\,(\mathbf{u}-\mathbf{v})}{dt} .$$

The coefficient of the associated mass C_m is independent of the Reynolds number and is equal to 1/2. The above formula has been theoretically proved for a potential flow of an ideal incompressible liquid around a particle and a creeping flow past a particle [8].

In the case of a relative accelerated flow past a particle, the equation of motion of the particle is considered in a noninertial coordinate system. The inertia force $\rho d\mathbf{u}/dt$ is added to the external body forces, and this leads to the isolation of the Archimedes force [8]:

$$\mathbf{f}_\mathrm{A} = \rho V_\mathrm{p} \frac{d\mathbf{u}}{dt} = \rho V_\mathrm{p} \left[\frac{\partial \mathbf{u}}{dt} + (\mathbf{u} \cdot \nabla)\, \mathbf{u} \right].$$

The time derivative is taken relative to a fixed inertial coordinate system. The Archimedes force is defined as the force of the inviscid origin (the derivative $d\mathbf{u}/dt$ is expressed in terms of the pressure gradient when using the Euler equation). The replacement of the pressure gradient by a more general relation which follows from the Navier–Stokes equations leads, under certain conditions, to a situation where, depending on the magnitude and sign of $\Delta\mathbf{u}$, the relative velocity of liquid and particle may take values from $-\infty$ to $+\infty$ [9].

The Bassé force (the force of viscous recovery) takes into account the influence of the prehistory of the particle motion (because of the nonstationary state of the viscous mixing layer around the particle) and the additional resistance to the motion of the particle from the liquid because of the change in its relative velocity:

$$\mathbf{f}_\mathrm{B} = 6 r_\mathrm{p}^2 (\pi \rho \mu)^{1/2} \int_0^t \frac{d\,(\mathbf{u}-\mathbf{v})}{d\tau} \frac{d\tau}{(t-\tau)^{1/2}} .$$

At high Reynolds numbers nonlinear inertia effects prevail in the relative motion of the particle and liquid, whereas nonstationary effects in the gas phase are usually not taken into account.

Subject to the above-given relations, the equation of the motion of the particle takes the form (the Bassé force is not included to simplify the form of the equation)

$$\frac{d\mathbf{v}}{dt} = \frac{1}{\gamma + C_m} \frac{3C_D}{8r_\mathrm{p}} \left| \mathbf{u}-\mathbf{v} \right| (\mathbf{u}-\mathbf{v}) + \frac{C_L}{\gamma + C_m} (\mathbf{u}-\mathbf{v}) \times \boldsymbol{\Omega} + \frac{1 + C_m}{\gamma + C_m} \frac{d\mathbf{u}}{dt} + \frac{\gamma - 1}{\gamma + C_m} \mathbf{g} . \tag{2}$$

The kinematic relation

$$\frac{d\mathbf{r}_\mathrm{p}}{dt} = \mathbf{v} \tag{3}$$

allows one to calculate the radius-vector of the center of masses of the particle.

Contribution of Various Force Factors. We assume that the vorticity has only one component Ω_x, which is perpendicular to the plane in which the particle moves translationally. The ratio of the magnitude of the buoyancy force to the magnitude of the resistance force is of the order of the rotational Reynolds number:

$$\frac{f_L}{f_D} \sim \mathrm{Re}_\omega ,$$

where $\mathrm{Re}_\omega = r_\mathrm{p}^2\omega/6\nu$. As the characteristic velocity we adopt the linear velocity of the points of the meridional section of the sphere ωr_p. At $\mathrm{Re}_\omega << 1$ the displacement of particles due to the action of the buoyancy force is not large.

Let U and t_v denote the characteristic velocity of the particle in the relative motion and the characteristic time of the change in the relative velocity of the particle. With allowance for the fact that

$$|\mathbf{u}-\mathbf{v}| \sim U\,, \quad \int_0^t \frac{d(\mathbf{u}-\mathbf{v})}{d\tau}\frac{d\tau}{(t-\tau)^{1/2}} \sim \frac{U}{t_v}\int_0^{t_v}\frac{d\tau}{(t_v-\tau)^{1/2}} = \frac{2U}{t_v^{1/2}}\,,$$

it is not difficult to obtain the following estimates:

$$\frac{f_\mathrm{B}}{f_D} \sim \frac{f_m}{f_\mathrm{B}} \sim r_\mathrm{p}\left(\frac{\rho}{\mu t_v}\right)^{1/2} = \frac{1}{2}\left(\mathrm{Re_p Sh_p}\right)^{1/2}, \quad f_m \sim f_\mathrm{A}\,,$$

where

$$\mathrm{Re_p} = \frac{2r_\mathrm{p}\,\rho U}{\mu}\,; \quad \mathrm{Sh_p} = \frac{2r_\mathrm{p}}{Ut_v}\,.$$

As the characteristic time we adopt the time of dynamic relaxation:

$$t_v = \frac{8r_\mathrm{p}\,\rho_\mathrm{p}}{3C_D\rho\,|\mathbf{u}-\mathbf{v}|}\,,$$

then

$$\frac{f_\mathrm{B}}{f_D} \sim \frac{f_m}{f_\mathrm{B}} \sim \left(\frac{\rho}{\rho_\mathrm{p}}\right)^{1/2}.$$

When $\rho << \rho_\mathrm{p}$, the following inequalities hold:

$$f_\mathrm{A} \sim f_m << f_\mathrm{B} << f_D\,.$$

The Archimedes force has the same order of smallness as the force of the associated mass (the components $\mathbf{f}_\mathrm{A}$ and $\mathbf{f}_m$ should be taken into account or disregarded simultaneously). The ratio of the forces $\mathbf{f}_\mathrm{A}$, $\mathbf{f}_m$, and $\mathbf{f}_\mathrm{B}$ to the resistance force $\mathbf{f}_D$ has the order of the phase density ratio ρ/ρ_p; therefore the nonstationary effects of the interaction of phases can be neglected if $\rho << \rho_\mathrm{p}$. The exception is provided by the motion of a particle in a flow with high velocity gradients (for example, in transition of the particle through the compression shock), when the time of relaxation is specified not by the retardation of the particle in a homogeneous flow (the gas velocity in the vicinity of the particle does not change), but rather by the change in the relative velocity of the particle due to the change in the gas phase velocity.

At high Reynolds numbers the prevailing influence is exerted by nonlinear inertia effects, whereas the influence of nonstationary effects (often defined as hereditary ones) turns out to be small in the gas phase. The influence of nonstationary effects should be taken into account if the local accelerations of the medium (the Archimedes force) or the differences between the local acceleration of the medium and particle (the force of the associated mass and the Bassé force) are great. Then, the estimates obtained require correction by taking into account the local acceleration of the liquid and particle.

Flow Between Rotating Coaxial Cylinders. We will consider the flow of a viscous incompressible liquid between the surfaces of two uniformly rotating (with angular velocities ω_1 and ω_2) coaxial horizontal cylinders of radii r_1 and r_2.

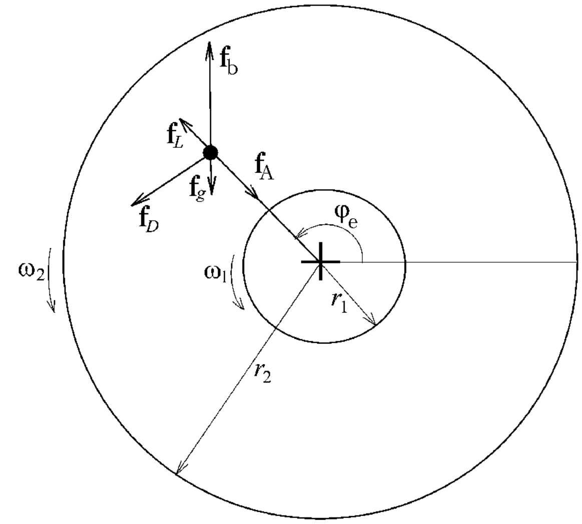

Fig. 1. Forces acting on the particle located in the gap between two coaxial horizontal cylinders.

It is thought that the motion of the liquid remains purely rotational with the axial velocity component equal to zero. The end effects connected with the finite length of the cylinders are not taken into account. A change in the velocity occurs only as a result of the action of friction forces between neighboring cylindrical layers of the liquid, and the velocity depends only on the distance to the symmetry axis $u_r = 0$ and $u_\varphi(r)$. The streamlines are concentric circles located in the plane perpendicular to the common axis of the cylinders.

The equation of liquid motion in polar coordinates has the form [10]

$$\frac{\rho u_\varphi^2}{r} = \frac{\partial p}{\partial r}, \tag{4}$$

$$\frac{\partial^2 u_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r^2} = 0 . \tag{5}$$

The boundary conditions are set on the surface of the inner and outer cylinders

$$u_\varphi = r_1\omega_1 \text{ at } r = r_1 , \quad u_\varphi = r_2\omega_2 \text{ at } r = r_2 .$$

Equation (4) shows that the radial change in pressure yields a force needed for preserving the motion of liquid along the circular trajectories. Equation (5) represents the equality between the rate of increase in the moment of momentum of the cylindrical layer of liquid and the moment of the resultant pair of friction forces on its inner and outer surfaces. The solution of Eq. (5) has the form [10]

$$u_\varphi(r) = \frac{c_1}{r} + c_2 r , \quad c_1 = \frac{(\omega_1 - \omega_2)\, r_1^2 r_2^2}{r_2^2 - r_1^2} , \quad c_2 = \frac{\omega_2 r_2^2 - \omega_1 r_1^2}{r_2^2 - r_1^2} , \tag{6}$$

whence we can obtain velocity distribution in various specific cases. Assuming that $r_1 = 0$ for the flow inside a rotating cylinder, we have $u_\varphi = \omega_2 r$, which corresponds to the rotation of a liquid as a solid body when all tangent stresses are equal to zero [10]. In the case of a single cylinder rotating in an infinite mass of liquid (when $r_2 \to \infty$), formula (6) takes the form $u_\varphi = r_1^2\omega_1/r$. The velocity field in the vicinity of the cylinder is the same as in the vicinity of a vortex filament with the stress $\Gamma = 2\pi r_1^2\omega_1$ which rotates in liquid without friction [10].

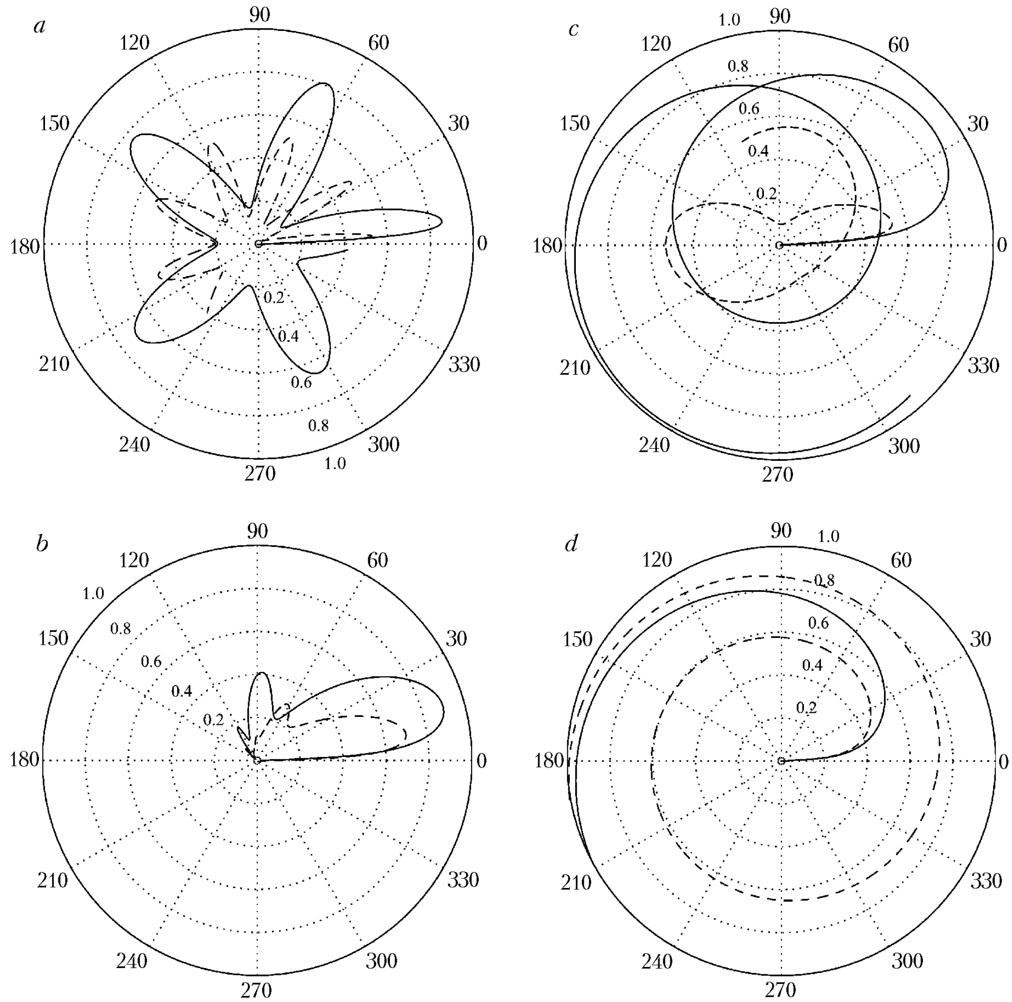

Fig. 2. Trajectories of the particle in the gap between two concentric rotating cylinders: a, b) $\gamma = 10^{-3}$, $r_2 = 1.0$; c, d) 10^3 and 5.0 m [a, c) $\mathrm{Re}_\omega = 4.16$; b, d) 0.416].

With allowance for the diagram of forces (Fig. 1) acting on the particle, Eq. (2) is written as

$$\frac{dv_r}{dt} - \frac{v_\varphi^2}{r} = \frac{C_L}{\gamma + C_m}(u_\varphi - v_\varphi)\,\Omega_x - \frac{1 + C_m}{\gamma + C_m}\frac{u_\varphi^2}{r} - \frac{\gamma - 1}{\gamma + C_m}\,g\sin\varphi\,,$$

$$\frac{dv_\varphi}{dt} + \frac{v_r v_\varphi}{r} = \frac{\gamma}{\gamma + C_m}\frac{u_\varphi - v_\varphi}{t_v} - \frac{\gamma - 1}{\gamma + C_m}\,g\cos\varphi\,.$$

At the instant of time $t = 0$ the initial conditions have the form

$$v_r = 0\,,\quad v_\varphi = r_1\omega_1\,,\quad r = r_1\,,\quad \varphi = 0\,.$$

Some results of calculations that illustrate the characteristic features of the motion of light and heavy particles are demonstrated in Fig. 2. The calculations were carried out at the following values of the parameters: $r_\mathrm{p} = 5\cdot10^{-4}$ m, $\gamma = 10^{-3}$–10^3, $\omega = 100$ 1/sec, and $\nu = 10^{-6}$–10^{-4} m^2/sec. The Reynolds number was varied by a respective change in viscosity. Integration with respect to time was carried out over the interval (0, t_f] either up to the specified instant

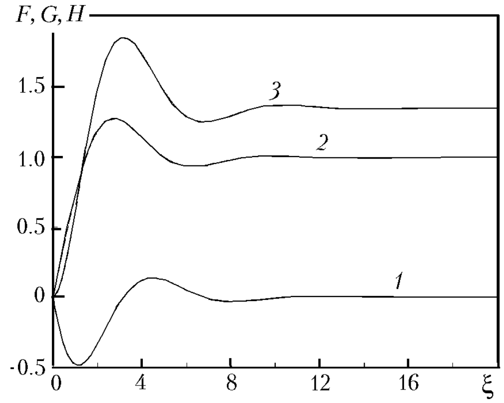

Fig. 3. Velocity distribution in the boundary layer formed during liquid rotation near a fixed base: 1) F; 2) G; 3) H.

of time or to the instant of time corresponding to the fall down of the particle onto the surface of the inner or outer cylinder. The dashed line corresponds to the particle trajectory obtained without allowance for the influence of buoyancy force. In the case of light particles and low viscosity of the carrying flow, the particle moves along the cycloid, i.e., its trajectory has equilibrium points (Fig. 2a) at which $v_r = v_\varphi = 0$ (here $t_f = 0.115$ sec). An increase in the viscosity (a decrease in Re_ω) leads to a decrease in the amplitude of the cycloid lobes and to the fall of the particle onto the surface of the inner cylinder (Fig. 2b). The settling of the particle occurs at the instant of time $t_f = 0.102$ sec.

Assuming that $v_r = v_\varphi = 0$, we find the equilibrium position of the particle:

$$\tan \varphi_e = 2\left[2C_L - (1 + C_m)\right] \mathrm{Re}_\omega = -\,\mathrm{Re}_\omega\,, \quad r_e = \frac{(\gamma - 1)\, g \sin \varphi_e}{[2C_L - (1 + C_m)]\, \omega^2} = (1 - \gamma)\, \frac{2g}{\omega^2} \sin \varphi_e\,.$$

It is assumed that $C_L = C_m = 1/2$.

The trajectory of the heavy particle resembles an untwisting spiral (Fig. 2c, d) whose number of turns depends on the rotational Reynolds number. The time of integration in these figures is $t_f = 0.2201$ sec (the settling of the particle onto the surface of the outer cylinder occurs at $t_f = 0.2999$ sec) and $t_f = 0.0622$ sec, respectively. In both cases, the particle settles onto the surface of the outer cylinder. Neglect of the buoyancy force leads to an increase in the time of settling of the particle up to 0.1 sec (the dashed curve in Fig. 2d). During the motion of both light and heavy particles, disregard of the action of the buoyancy force leads to substantial distortion of the results of numerical modeling.

Rotational Motion of Liquid over a Plane. We will consider a vortex flow originating near a fixed plane wall in the case where at a large distance from its surface the liquid rotates with a constant angular velocity [12].

For the particles of the liquid located at a large distance from the wall the centrifugal force and the radial pressure gradient are mutually equilibrated. For the particles of the liquid located near the wall, the circumferential velocity was lowered because of the retardation; therefore here the centrifugal force is reduced considerably, while the radial pressure gradient directed inward remains the same as at a great distance from the wall. Near the wall, an inward-directed radial flow originates which, due to the continuity condition, causes an ascending flow in the axial direction.

We will superpose the plane $x = 0$ with the fixed wall. We assume that at a great distance from the wall the liquid rotates as a solid body with angular velocity ω. Having introduced the dimensionless coordinate $\xi = x(\omega/\nu)^{1/2}$, the velocity distribution is written as

$$u_x = (\nu\omega)^{1/2} H(\xi)\,, \quad u_r = r\omega F(\xi)\,, \quad u_\varphi = r\omega G(\xi)\,.$$

The equations that describe the flow have the form [12]

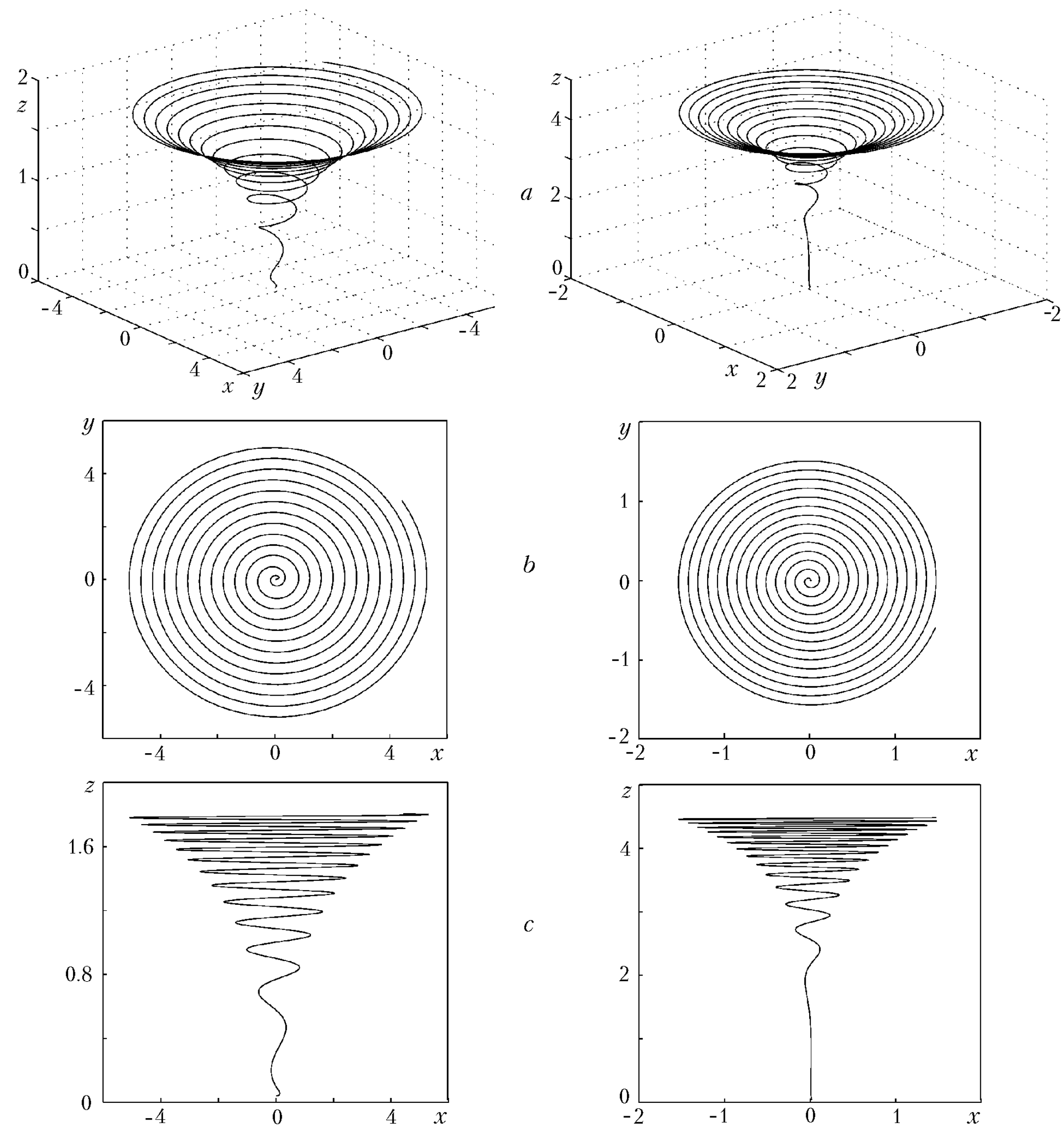

Fig. 4. Trajectories of the particle in a rotational liquid flow above a fixed base at r_p = $8.55 \cdot 10^{-6}$ m, Stk = 2.8, t_f = 0.04 sec (on the left) and r_p = $2.28 \cdot 10^{-6}$ m, Stk = 0.2, t_f = 0.07 sec (on the right). x, y, z, m.

$$2F + H' = 0\,, \quad F^2 - G^2 + HF' - F'' + 1 = 0\,, \quad 2GF + HG' - G'' = 0\,. \tag{7}$$

The boundary conditions are set on the wall and in the external flow:

$$F = 0\,, \quad G = 0\,, \quad H = 0 \quad \text{at} \quad \xi = 0\,; \quad F = 0\,, \quad G = 1 \quad \text{at} \quad \xi = \infty\,.$$

The distribution of the velocity of the carrying flow is calculated by numerical integration of the system of equations (7). The liquid velocity at the points lying on the particle trajectory is found with the aid of spline-interpolation. The distribution of the velocity components are shown in Fig. 3. Conjugation of the solution with the boundary condition at infinity occurs at $\xi \sim 14$. Up to a certain height the velocity increases in all directions. The axial velocity component is everywhere positive (an ascending motion of liquid occurs). Near the wall the radial velocity component is directed inward (to the rotation axis). At a great height the ascending flow damps out, since radial motion of liquid outside occurs there.

The trajectories of the particle in the form of an untwisting spiral are shown in Fig. 4a; Fig. 4b and c presents the projections of the particle trajectory onto the coordinate planes. The calculations were performed at $\gamma = 10^{-3}$–10^{3}, $\omega = 500$ 1/sec, and $\mu = 1.71 \cdot 10^{-5}$ kg/(m·sec). The Stokes number was varied due to the change in the particle size.

A change in the particle size and allowance for the gravity force do not lead to a qualitative change in the pattern of particle motion. A change in the parameters of the problem leads to an increase in the amplitude of the vibrations of the particle relative to the rotation axis.

Conclusions. The modeling of the motion of a sample particle in flows with concentrated vorticity is carried out. As examples, the motion of a particle in the gap between concentric rotating cylinders and the involvement of the particle in the rotational motion of the liquid over a fixed base are considered. During the motion of both light and heavy particles, disregard of the buoyancy force leads to a substantial distortion of the results of numerical modeling.

The results of calculations can be applied for modeling two-phase flows with allowance of the inverse effect of an impurity and visualization of the rotational motion of liquid.

NOTATION

c, integration constant; *C*, coefficient; **f**, force acting on the particle, N; *F*, dimensionless radial velocity; **g**, free-fall acceleration, m/sec^2; *G*, dimensionless circumferential velocity; *H*, dimensionless axial velocity; **l**, vector specifying the direction of integration; *m*, mass, kg; *p*, pressure, Pa; *r*, radius, m; Re, Reynolds number; *S*, area, m^2; Sh, Strouhal number; Stk, Stokes number; *t*, time, sec; **u**, vector of the carrying flow velocity, m/sec; *U*, characteristic velocity, m/sec; **v**, vector of particle velocity, m/sec; *V*, volume, m^3; *x*, *y*, *z*, Cartesian coordinates, m; γ, ratio of particle density to the carrying flow density, m/sec; Γ, circulation, m^2/sec; μ, dynamic viscosity, kg/(m·sec); ν, kinematic viscosity, m^2/sec; ξ, dimensionless axial coordinate; ρ, density, kg/m^3; σ, tensor of viscous stresses; τ, integration variable; φ, polar angle; ψ, function; ω, angular velocity, 1/sec; $\boldsymbol{\Omega}$, vorticity, 1/sec. Subscripts: b, buoyancy; *D*, resistance force; e, equilibrium; f, finite instant of time; *L*, lifting force; p, particle; 0, vortex core; 1 and 2, inner and outer cylinders.

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